Journal of
Mechanical
Science and
Technology

Journal of Mechanical Science and Technology 21 (2007) 860~864

Stability of Periodic Motion on the Rotor-Bearing System with Coupling Faults of Crack and Rub-Impact

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(Manuscript Received November 16, 2006; Revised January 8, 2007)

Abstract

A dynamic model was set up for the rotor–bearing system with coupling faults of crack and rub-impact. Using the continuation-shooting algorithm for periodic solution of nonlinear non-autonomous system, the stability of the system periodic motion was studied by the Floquet theory. The saddle-node bifurcation, periodic-doubling bifurcation and the Hopf bifurcation were found. There are peculiar dynamic characteristics of the rotor–bearing system with coupling faults of crack and rub-impact that differs from single fault. The conclusions provide theoretic basis reference for the failure diagnosis of the rotor-bearing system with coupling faults of crack and rub-impact fault.

Keywords: Rotor-bearing system; Crack and rub-impact; Floquet theory

1. Introduction

The crack of rotor and rub-impact of rotor-stator are the frequent faults of rotor-bearing system in rotating machinery, which can greatly do harm to safe operation of mechanical system. Lee (1992) proved the correctness of the crack diagnosis using numerical imitates method and experimental verification. Gasch (1993) built a widely applicable nonlinear dynamics model of rotor-bearing system with cracking-shaft on the basis of transverse crack expanding theory and taking the distortion and whirl of the shaft into account. Huang (1993) studied the stability of a rotating shaft containing a transverse crack. Papadopoulos (1994) studied the torsional vibration of rotors with transverse surface cracks. Yang *et al.* (2001) analyzed the influence to the dynamics of the rotor system by the expanding of crack on the shaft. Some scholars (Ehrich, 1992 ; Chu *et al.*, 1998 ; LUO *et al.*, 2003) have made development research about bifurcation and chaos responses caused by the rub-impact fault of the rotor system. Their productions provide prodigious references on the rotating machinery fault diagnosis. In this paper, the coupling faults model of elastic rotor-bearing system with nonlinear oil film forces was set up. Stability of periodic motion on the rotor-bearing system with coupling faults of crack and rub-impact was analyzed by the continuation-shooting algorithm for periodic solution of nonlinear non-autonomous system and the Floquet theory. The results provide a theoretic basis reference for the fault diagnosis of the multi-fault rotor system.

2. Mechanical model and motion differential equation

The rotor-bearing system model and rub-impact force and the model of crack section are shown in Fig. 1. The system mass equivalently centralizes the center

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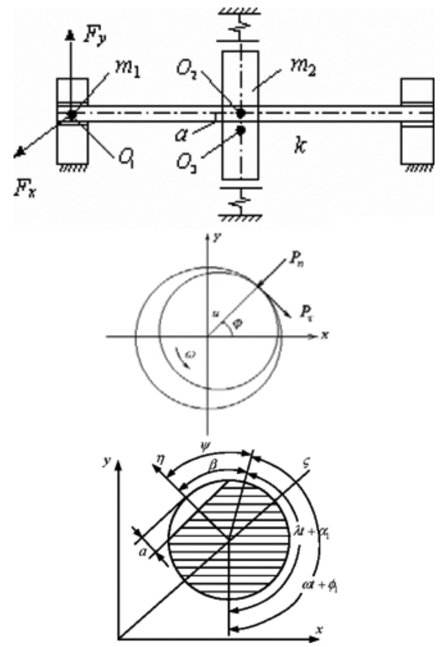


Fig. 1. Models of rotor-bearing system with coupling faults, rub-impact force and section of crack.

of disc and bearing support respectively. The torsion vibration and gyro moment are ignored. Only lateral vibration of system is considered. Both ends of rotor are supported by sliding bearings with symmetrical structures, O_1 is geometric centers of bearing, O_2 is geometric centers of rotor, O_3 is center of mass of rotor, m_1 is lumped masses of rotor at bearing, m_2 is equivalently lumped masses at disc. Elastic shaft with zero quality connect disc with bearing, k is stiffness of elastic axes. There is a transversal crack with deepness of a in the middle of shaft.

3. Rub-impact force

It is assumed that there is an initial clearance of δ_0 between the rotor and stator, when rub-impact fault occurs, the radial impact force and the tangential rub force can be expressed as

$$\begin{cases} P_n = (e-\delta_0)k_c \\ P_\tau = (f+bv)P_n \end{cases} \quad (e \ge \delta_0) \tag{1}$$

where f is the friction coefficient without regard to velocity influence, $v=\sqrt{\dot{x}^2+\dot{y}^2}$ is relative slip velocity between rotor and stator, $e=\sqrt{x^2+y^2}$ is the radial displacement of the rotor, k_c is rub-impact stiffness of rotor-stator. When rub-impact happens, the radial impact force and the tangential rub force can be written in x-y coordinate as

$$\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} = -\frac{(e-\delta_0)k_c}{e}\begin{bmatrix} 1 & -(f+bv) \\ (f+bv) & 1 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}, \ (e \ge \delta_0) \tag{2}$$

4. Differential equation of motion

Assuming the radial displacements in the bearing position are (x_1, y_1), the radial displacements of discs are (x_2, y_2), the differential equations of the system can be written as

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + k[1-\frac{\varepsilon\delta}{2}F(\psi)](x_1-x_2) + \frac{\varepsilon k}{2}F(\psi)[(x_1-x_2)\cos 2t + (y_1-y_2)\sin 2t] = F_x(x_1, y_1, \dot{x}_1, \dot{y}_1) \\ m_1\ddot{y}_1 + c_1\dot{y}_1 + k[1-\frac{\varepsilon\delta}{2}F(\psi)](y_1-y_2) + \frac{\varepsilon k}{2}F(\psi)[(x_1-x_2)\sin 2t - (y_1-y_2)\cos 2t] = F_y(x_1, y_1, \dot{x}_1, \dot{y}_1) - m_1 g \\ m_2\ddot{x}_2 + c_2\dot{x}_2 + 2k[1-\frac{\varepsilon\delta}{2}F(\psi)](x_2-x_1) + \varepsilon k F(\psi)[(x_2-x_1)\cos 2t + (y_2-y_1)\sin 2t] = P_x(x_2, y_2) + m_2 u\omega^2 \cos\omega t \\ m_2\ddot{y}_2 + c_2\dot{y}_2 + 2k[1-\frac{\varepsilon\delta}{2}F(\psi)](y_2-y_1) + \varepsilon k F(\psi)[(x_2-x_1)\sin 2t - (y_2-y_1)\cos 2t] = P_y(x_2, y_2) + m_2 u\omega^2 \sin\omega t - m_2 g \end{cases} \tag{3}$$

where ε and δ are relative stiffness parameter only about crack deepness of (Yang *et al.*, 2001), $F(\psi)$ is open and close function of crack:

$$F(\psi) = \frac{1+\cos\psi}{2}, \quad \psi = t - \phi_0 + \beta - \text{arctg}\frac{y}{x} \tag{4}$$

β is contained angle between crack direction and eccentricity; ϕ_0 is initial phase place; c_1 is damping coefficient at bearing; c_2 is damping coefficient of rotary disc; u is eccentricity amount of the rotor; ω is rotating speed. F_x and F_y are nonlinear oil film force component; the expression formula is shown in the reference by Luo *et al.* (2003). $P_x(x_2, y_2)$ and $P_y(x_2, y_2)$ are rub-impact force components of rotor systems at x and y directions respectively, they are shown as Eq. (2).

5. Stability analysis of the system periodic motion

5.1 Solution method of periodic motion and stability of system

The dynamic problem of unbalanced rotor-bearing system can be summed up the following non-automatic system(Liu *et al.* 1999):

$$\dot{\mathbf{X}} = \mathbf{F}(\mathbf{X}, t, \omega) \qquad (\mathbf{X}, t) \in \mathbf{R}^n \times \mathbf{R} \tag{5}$$

$\mathbf{X}$ is state vector of system. By Poincaré mapping, the periodic solution and stability of continuous dynamic system can be transformed as solving stationary point of Poincaré mapping and judging stability problem. Namely solving zero solution of the following nonlinear equation group.

$$\mathbf{G}(\mathbf{X},\omega)=\mathbf{X}-\mathrm{P}(\mathbf{X},\omega)=\mathbf{0} \tag{6}$$

where P is Poincaré mapping operator.

Initial iterative value is determined by solving curve $\mathbf{X}=\mathbf{X}(\omega)$ to determine, namely solving initial value question of the following ordinary differential equation:

$$\begin{cases}\dot{\mathbf{X}}(\omega)=-[\mathbf{G}'_{\mathbf{X}}(\mathbf{X},\omega)]^{-1}\cdot G'_{\omega}(\mathbf{X},\omega)\\ \mathbf{X}(\omega_0)=\mathbf{X}_0\end{cases} \tag{7}$$

Considering computational efficiency, the midpoint integral formula is selected to solve above initial value problem:

$$\begin{cases}\mathbf{X}_1=\mathbf{X}_0-\Delta\omega[\mathbf{G}'_{\mathbf{X}}(\mathbf{X}_0,\omega)]^{-1}\cdot\mathbf{G}'_{\omega}(\mathbf{X}_0,\omega)\\ \mathbf{X}_{k+1/2}=\mathbf{X}_k+\dfrac{1}{2}(\mathbf{X}_k-\mathbf{X}_{k-1})\\ \mathbf{X}_{k+1}=\mathbf{X}_k-\Delta\omega[\mathbf{G}'_{\mathbf{X}}(\mathbf{X}_{k+1/2},\omega)]^{-1}\cdot\mathbf{G}'_{\omega}(\mathbf{X}_0,\omega)\end{cases} \tag{8}$$

$\mathbf{X}_k$ is periodic solution obtained by k steps Newton iteration , $\Delta\omega$ is parameter step. Damping factor λ is introduced to the algorithm to improve Newton iteration form.

$$\begin{cases}\mathbf{X}_{k+1}=\mathbf{X}_k+\lambda\Delta\mathbf{X}, \quad 0\le\lambda\le 1\\ \mathrm{J}\cdot\Delta\mathbf{X}=-\mathrm{G}\end{cases} \tag{9}$$

J is Jacobi matrix. Defining scalar function $f(\mathbf{X})$, if

$$f(\mathbf{X})=\frac{1}{2}\|G(\mathbf{X})\|^2 \tag{10}$$

Firstly, taking $\lambda=1$, if $\mathbf{X}_{k+1}$ computed by equation 8 is drop point of $f(\mathbf{X})$, here the method is turned into Newton-Raphson method, which has second order rapidity of convergence. Otherwise, λ is gradually reduced so as to find drop point. In order to make dropping velocity of step lengths to coincide with f, judging $\mathbf{X}_{k+1}$ is or isn't drop point of f by the following formula:

$$f(\mathbf{X}_{k+1})\le f(\mathbf{X}_k)+\alpha\nabla f(\mathbf{X}_{k+1}-\mathbf{X}_k) \tag{11}$$

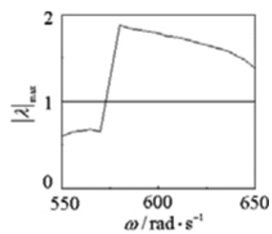
α is given a small positive number, generally $\alpha=10^{-4}$.

The method keeps second order rapidity of convergence and improves dependence on original value, so it is an effectual algorithm of convergence. State transfer matrix corresponding to periodic solution will be solved by iteration. The matrix characteristic values are called as Floquet multiplier. The stability of periodic solution is analyzed and the types of bifurcation are determined based on the position of Floquet multiplier in complex plane.

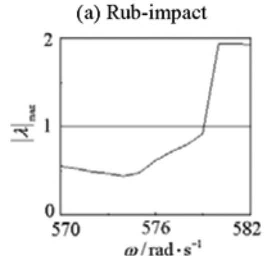
5.2 Bifurcation analysis and stability of rotor system with faults

The parameters of system are: m_1=4.0kg, m_2=32.1kg, R=25mm, L=12mm, c=0.11mm, $\mu=0.018\mathrm{P_a}\cdot\mathrm{s}$. β=0.0, $\phi_0=0$ u=0.05mm, c_1=1050 N·s/m, c_2=2100N·s/m, k =2.5×107N/m k_c=4.5×107 N/m $f=0.1$ $b=0.01$. The non-dimensional depth of crack is a=1.0(a=a/R).

Unbalanced response of system is tracked and periodic solution of system at different rotating speeds is obtained by shooting algorithm. The changing diagram of the Floquet multipliers of the rotor response with run-impact fault is shown in Fig. 2(a), it can be seen that when the rotary speed ω<571rad/s, the dominant Floquet multiplier is in unit circle of complex plane, so the periodic solution of system is gradually steady; when the rotating speed ω=571rad/s, the Floquet multiplier of system periodic solution crosses the unit circle by (+1, 0), the saddle-node bifurcation occurs based on Floquet theory, the response of system bifurcates to quasi-periodic motion from periodic-1. Figure 3 are the Poincaré maps and trajectory of the rotor responses when ω=605rad/s, which testify that the response of the system is quasi-periodic motion.

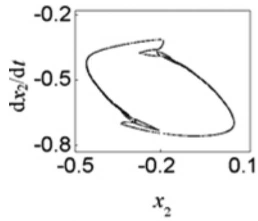


(a) Rub-impact

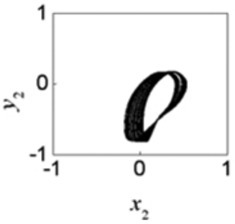


(b) Crack and rub-impact

Fig. 2. The changing of the Floquet multipliers of the rotor response.

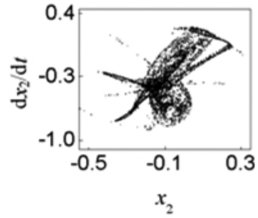


(a) Poincaré maps

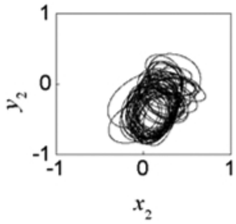


(b) Trajectory

Fig. 3 The responses of the rotor when ω=605rad/s.



(a) Poincaré maps

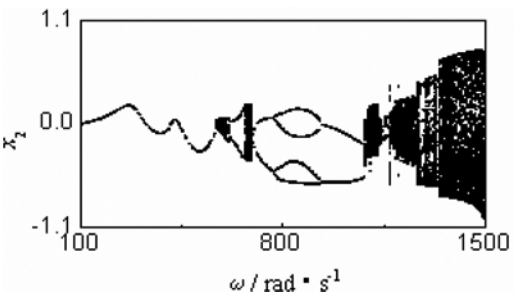


(b) Trajectory

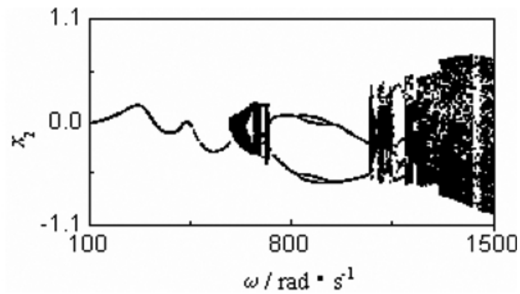
Fig. 4. The responses of the rotor when ω=1218rad/s.

Table 1. Characteristics of the rotor system with rub-impact fault at different ω.

ω (rad/s)	$\lvert\lambda\rvert_{max}$	λ_1, λ_2	Results
560	0.6642	0.4391+0.4983 *i*	periodic solution gradually steady (periodic-1)
571	1.8990	1.8990+0.0000 *i*	saddle-node bifurcation (periodic-1 → quasi-periodic motion)
780	1.0787	-1.0787+0.0000 *i*	periodic-doubling bifurcation (periodic-2 → periodic-4)
930	1.0319	-1.0319+0.0000 *i*	periodic-doubling reverse bifurcation (periodic-4 → periodic-2)
1105	1.3153	-1.1866+0.5674 *i*	Hopf bifurcation (periodic-2 → chaos)



(a) Rub-impact



(b) Crack and rub-impact

Fig. 5. The bifurcation diagrams of rotor-bearing system with ω.

The influence on stability of system periodic motion with rub-impact fault is further studied. Table 1 shows the characteristics of the rotor system with rub-impact fault at different ω, it can be seen that when ω=780rad/s, the Floquet multiplier of periodic solution of system cross the unit cycle by (-1,0), the periodic-doubling bifurcation occurs based on Floquet theory, form period-2 bifurcation to period-4 motion. when ω=930rad/s, the Floquet multiplier of periodic solution of system cross the unit cycle by (-1,0), the periodic-doubling reverse bifurcation occurs based on Floquet theory, form periodic-4 bifurcation to periodic-

2 motion. when ω=1105rad/s, the Floquet multiplier of system periodic solution crosses the unit circle by a pair conjugate complex numbers, Hopf bifurcation occurs based on Floquet theory, the response of system bifurcates quasi-periodic motion from periodic-2. Figure 4 are the Poincaré maps and trajectory of the rotor responses when ω=1218rad/s, which testify that the response of the system is chaos motion. Figure 5(a) shows the bifurcation map of system response numerically integrated by the forth order Runge-Kutta method with define step which verifies the correctness of the results by shooting method.

The changing diagram of the Floquet multipliers of the rotor response with coupling faults of crack and rub-impact when a=1.0 is shown in Fig. 2 (b). It can be seen that when the rotary speed ω<560rad/s, the dominant Floquet multiplier is in unit circle of complex plane, so the periodic solution of system is gradually steady; when the rotating speed ω=560rad/s, the Floquet multiplier of system periodic solution crosses the unit circle by (+1, 0), the saddle-node bifurcation occurs based on Floquet theory, the response of system bifurcates to quasi-periodic motion from periodic-1.

Fig. 5(b) shows the bifurcation map of system response numerically integrated by the forth order Runge-Kutta method with define step which verifies the correctness of the results by shooting method. Comparing with Fig. 2(a), to rotor-bearing system with rub-impact fault, system unsteady with saddle-node bifurcation both rub-impact fault and coupling faults. The influence of crack fault reduces the unsteady rotating speeds, namely stability of the rotor system with coupling faults is easy to unsteady.

6. Conclusions

(1) Nonlinear dynamic model of rotor-bearing system with crack and rub-impact coupling faults is set up, steady periodic solution of system is computed by continuation-shooting algorithm for periodic solution of nonlinear non-autonomous system, stability of system periodic motion and unsteady law are discussed by Floquet theory.

(2) The rub-impact rotor system unsteady with saddle-node bifurcation, and there are different forms of periodic-doubling bifurcation, periodic-doubling reverse bifurcation and Hopf bifurcation at different rotating speeds.

(3) For rotor-bearing system with crack and rub-impact coupling faults, the main influence to the nonlinear characteristics is rub-impact, the influence of crack fault reduces the unsteady rotating speeds, the stability of the rotor system with coupling faults is easy to unsteady. The conclusions provide theoretic basis reference for the failure diagnosis of the rotor system with coupling faults of crack and rub-impact.

Acknowledgements

This research is supported financially by National Natural Science Foundation of China(Grant No. 50535010) and (No. 50275024)

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